

## Notation

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The infinite sequence  $\{a_1, a_2, a_3, a_4, \dots\}$  may also be represented as

$$\{a_n\} \text{ or } \{a_n\}_{n=1}^{\infty}$$

## Definitions

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### *Limit of a Sequence, Convergent, Divergent*

- The **limit**,  $L$ , of a sequence  $\{a_n\}$  is  $L = \lim_{n \rightarrow \infty} a_n$ 
  - ▷ If  $L$  exists, then  $\{a_n\}$  is **convergent**.
  - ▷ If  $L$  doesn't exist, then  $\{a_n\}$  is **divergent**.

### *Monotonic Sequences*

- A sequence is **monotonic** if its elements are always non-increasing or non-decreasing.
  - ▷ If  $a_n \geq a_{n+1}$  for all  $n$ , the sequence is **monotonically decreasing**  
Each item is smaller than the one before it.
  - ▷ If  $a_n \leq a_{n+1}$  for all  $n$ , the sequence is **monotonically increasing**  
Each item is larger than the one before it.

### *Bounded Sequences*

Given real numbers  $M$  and  $N$ , a sequence is

- **Bounded above** if  $a_n \leq M$  for all  $n$ .  
That is, none of the sequence's elements are greater than  $M$ .
- **Bounded below** if  $a_n \geq N$  for all  $n$ .  
That is, none of the sequence's elements are less than  $N$ .
- **Bounded** if it is bounded both above and below.  
That is, all of the sequence's elements are between  $M$  and  $N$ , inclusive

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## Infinite Sequence Theorems

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### *Absolute Value Theorem*

- If  $\lim_{n \rightarrow \infty} |a_n| = 0$ , then  $\lim_{n \rightarrow \infty} a_n = 0$

### *Monotonic Bound Sequences Theorem*

- If  $\{a_n\}$  is monotonic and bound, then it converges.
- If  $\{a_n\}$  is monotonically increasing and bound above, then it converges.
- If  $\{a_n\}$  is monotonically decreasing and bound below, then it converges.

### *Geometric Sequences Theorem*

- $\{r^n\}$  converges if  $-1 < r \leq 1$ .

### *Power Law*

- $\lim_{n \rightarrow \infty} a_n^p = [\lim_{n \rightarrow \infty} a_n]^p$

### *Squeeze Theorem for Sequences*

Given three sequences  $a_n$ ,  $b_n$ , and  $c_n$ ,

- If:
  - ▷  $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n = L$ , and
  - ▷ There exists an integer  $N$  such that

$$a_n \leq c_n \leq b_n$$

for all  $n > N$

- Then:
  - ▷  $\lim_{n \rightarrow \infty} c_n = L$

### **Squeeze Theorem**

*English Language Version*

**if**

- $a_n$  and  $b_n$  both converge on  $L$
- all of  $c_n$ 's terms are between the corresponding terms of  $a_n$  and  $b_n$

**then**

- $c_n$  also converges on  $L$